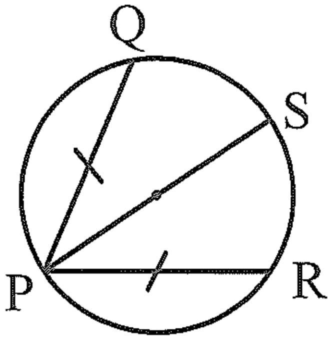
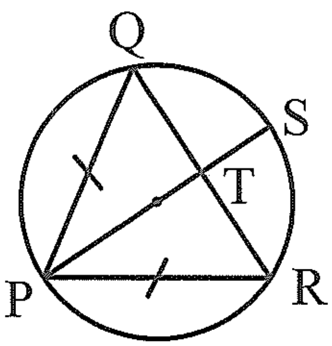
14. (a)



✓

(b) (i) In ΔPQS and ΔPRS

$\overline{PQ} \cong \overline{PR}$	Given	
$\overline{SP} \cong \overline{SP}$	Common	
$\angle PQS \cong \angle PRS$	Angle in Semi–circle Theorem	✓
$\therefore\ \Delta PQS \cong \Delta PRS$	RHS	✓
$\therefore\ QS = RS$	Congruent sides	✓



(ii) In ΔQPT and ΔRPT

$\angle QPT \cong \angle RPT$	Proven previously	✓
$\overline{PT} \cong \overline{PT}$	Common	
$\overline{PQ} \cong \overline{PR}$	Given	
$\therefore\ \Delta QPT \cong \Delta RPT$	SAS	✓
$\therefore\ \overline{QT} \cong \overline{RT}$	Congruent sides	✓
$\therefore\ \overline{SP}$ bisects $\overline{QR}$		[7]

20. (a) $\binom{9}{6} = 84$ ✓

(b) (i) $6! = 720$ ✓

(ii) $3! \times 4! = 144$ ✓✓ [4]

Assume $x+2$ is odd Now, if x is even $x = 2n$.

$\therefore\ x+2 = 2n+2$

$= 2(n+1)$

which is ~~even~~ not odd

$\therefore$ contradiction hence $x+2$ is even

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Question 13 **(9 marks)**

The points A, B and C have position vectors $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 14\mathbf{i} - 3\mathbf{j}$ and $\mathbf{c} = -5\mathbf{i} + 2\mathbf{j}$.

(a) Determine the angle between vectors $\mathbf{b}$ and $\mathbf{c}$, giving your answer rounded to one decimal place. (2 marks)

Using CAS:

$170 \cdot 3^\circ$ ✓✓

(b) Find the position vector of point D which divides $\overline{AC}$ internally in the ratio $5:3$. (3 marks)

$$\tilde{r}_D = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \frac{5}{8}\left(\overrightarrow{AC}\right) ✓$$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \frac{5}{8}\begin{pmatrix} -8 \\ -2 \end{pmatrix} ✓$$

$$= \begin{pmatrix} -2 \\ 2 \cdot 75 \end{pmatrix} ✓$$

(c) Express the vector $\mathbf{b}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. (4 marks)

$$\tilde{b} = \lambda\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \mu\begin{pmatrix} -5 \\ 2 \end{pmatrix} ✓$$

$$\therefore 3\lambda - 5\mu = 14 ✓$$

$$4\lambda + 2\mu = -3$$

$$\therefore \lambda = 0 \cdot 5 \quad \mu = -2 \cdot 5 ✓$$

$$\therefore \tilde{b} = 0 \cdot 5\,\tilde{a} - 2 \cdot 5\,\tilde{c} ✓$$

Question 10 **(10 marks)**

Three points are given by $A(1, 2)$, $B(p,-2)$ and $C(12,4)$.

(a) Determine a unit vector parallel to the line through AC. (2 marks)

$$\overrightarrow{AC} = \begin{pmatrix} 11 \\ 2 \end{pmatrix}$$ ✓

$$\therefore \text{unit vector} = \frac{11\tilde{i} + 2\tilde{j}}{\sqrt{125}}$$ ✓

(b) Write down a vector equation of the line through AC. (1 mark)

$$\tilde{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 11 \\ 2 \end{pmatrix}$$ ✓

(c) Find the value of p if the lines through AB and BC are perpendicular and $p < 8$. (3 marks)

$$\overrightarrow{AB} = \begin{pmatrix} p-1 \\ -4 \end{pmatrix}, \quad \overrightarrow{BC} = \begin{pmatrix} 12-p \\ 6 \end{pmatrix}$$ ✓

$$\begin{pmatrix} p-1 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 12-p \\ 6 \end{pmatrix} = 0$$ ✓

$$(p-1)(12-p) - 24 = 0$$

$$\therefore p = 4, 9.$$

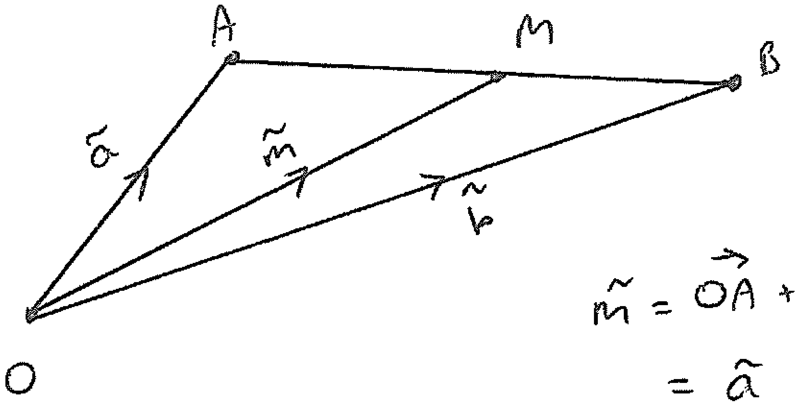
$$\therefore p = 4$$ ✓

Question 17 **(7 marks)**

M is the mid-point of line segment AB.

If $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OM}$ are $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{m}$ respectively

(a) Find an expression for $\underset{\sim}{m}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. (4)



$$\tilde{m} = \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AB}$$
$$= \tilde{a} + \tfrac{1}{2}(\tilde{b} - \tilde{a})$$
$$= \tfrac{1}{2}\tilde{a} + \tfrac{1}{2}\tilde{b}$$

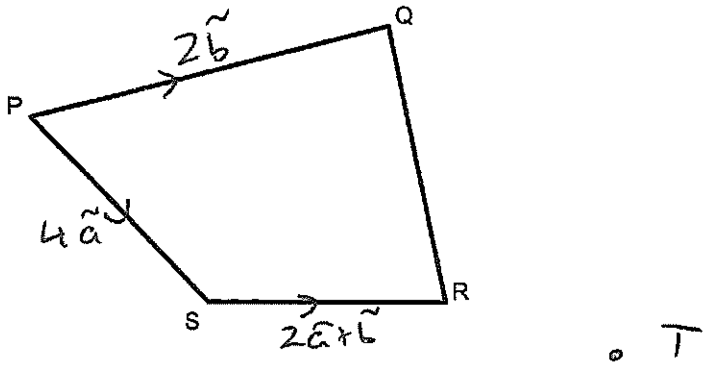
(b) Hence, or otherwise state the coordinates of S if S divides the line segment joining F(1,4) to G(6,9) in the ratio 1:1. (3)

$$S = \tfrac{1}{2}\begin{pmatrix}1\\4\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}6\\9\end{pmatrix}$$

$$\therefore S = (3\cdot5,\ 6\cdot5)$$

Question 13 **(6 marks)**

In the diagram $\overrightarrow{PQ} = 2\underset{\sim}{b}$, $\overrightarrow{PS} = 4\underset{\sim}{a}$ and $\overrightarrow{SR} = 2\underset{\sim}{a} + \underset{\sim}{b}$



(a) Express as simply as possible, in terms of $\underset{\sim}{a}$ and/or $\underset{\sim}{b}$ (2)

(i) $\overrightarrow{SQ}$ $-4\tilde{a} + 2\tilde{b}$ ✓

(ii) $\overrightarrow{QR}$ $6\tilde{a} - \tilde{b}$ ✓

There is another point, T.

(b) If $\overrightarrow{PT} = h\overrightarrow{PR}$, express $\overrightarrow{PT}$ in terms of h, $\underset{\sim}{a}$ and $\underset{\sim}{b}$ (1)

$\overrightarrow{PT} = h(6\tilde{a} + \tilde{b})$ ✓

or $6h\tilde{a} + h\tilde{b}$

(c) Given that $4\overrightarrow{ST} = \overrightarrow{SQ}$, calculate the value of h. (3)

$\overrightarrow{ST} = -4\tilde{a} + h(6\tilde{a} + \tilde{b})$ ✓

$\overrightarrow{SQ} = -4\tilde{a} + 2\tilde{b}$

$\therefore 4(-4\tilde{a} + h(6\tilde{a} + \tilde{b})) = -4\tilde{a} + 2\tilde{b}$ ✓

$\therefore -16 + 24h = -4$

$\therefore h = \frac{1}{2}$ ✓

Question 9 **(10 marks)**

(a) Point A has position vector $k\underset{\sim}{i} - \underset{\sim}{j}$. Point B has position vector $6\underset{\sim}{i} - k\underset{\sim}{j}$.

If $\left|\overrightarrow{AB}\right| = 5$, find the value(s) of k. (4)

$\overrightarrow{AB} = \begin{pmatrix} 6-k \\ -k+1 \end{pmatrix}$ ✓

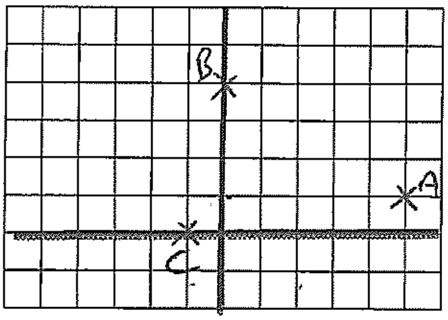
$5 = \sqrt{(6-k)^2 + (-k+1)^2}$ ✓

$K =$ [illegible] , ✓✓

(b) Let A= (5,1), B=(0,4), C=(-1,0)

Find

(Hint: Use the grid below to help you find the points)



(i) D such that $\overrightarrow{AB} = \overrightarrow{CD}$ ✓ ✓ (2)

D= (-6, 3)

(ii) F such that $\overrightarrow{AF} = -\overrightarrow{BC}$ ✓ ✓ (2)

F = (6, 5)

(iii) G such that $\overrightarrow{AB} = \overrightarrow{2GC}$ ✓ ✓ (2)

(1·5, -1·5)

Question 7 **(10 marks)**

(a) If $\underset{\sim}{a} = 6\underset{\sim}{i} - 4\underset{\sim}{j}$, $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$, $\underset{\sim}{c} = 2\underset{\sim}{i} + 5\underset{\sim}{j}$

(i) Determine $|\underset{\sim}{c} - \underset{\sim}{b}|$ Leave your answer as a surd (2)

$$\left|\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right| = \sqrt{2}$$

✓ ✓

(ii) Determine $2\underset{\sim}{b} - 3\underset{\sim}{a} + \underset{\sim}{c}$ (2)

$$\begin{pmatrix} 6 \\ 8 \end{pmatrix} - \begin{pmatrix} 18 \\ -12 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$ ✓

$$= \begin{pmatrix} -10 \\ 25 \end{pmatrix}$$ ✓

(iii) Determine a vector in the direction of $\underset{\sim}{a}$ but with a magnitude of 5. (3)

unit vector $\hat{a} = \dfrac{6\underset{\sim}{i} - 4\underset{\sim}{j}}{\sqrt{52}}$ ✓ ✓

$$\therefore \frac{5(6\underset{\sim}{i} - 4\underset{\sim}{j})}{\sqrt{52}}$$ ✓

(b) Find the value of k if $\underset{\sim}{p}$ and $\underset{\sim}{q}$ are parallel vectors.

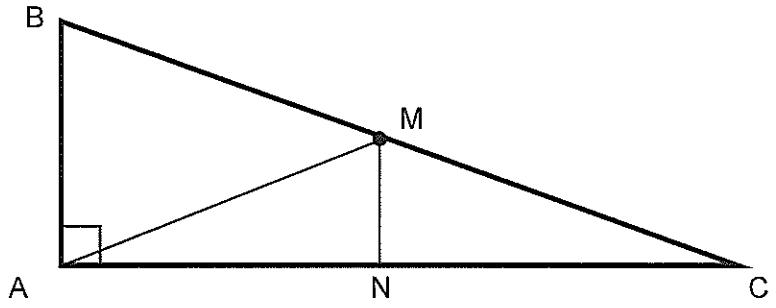
$$\underset{\sim}{p} = \sqrt{2}\begin{pmatrix} k \\ -3 \end{pmatrix}, \quad \underset{\sim}{q} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$ (3)

$$\frac{\sqrt{2}\,k}{2} = \sqrt{2}(-3)$$ ✓✓

$$\therefore k = -6$$ ✓

Question 15 **(4 marks)**

In the diagram below ABC is a right-angled triangle, and M is the mid-point of the hypotenuse BC.



Prove that M is equidistant from each of the vertices A, B and C.

Hint: Start by drawing the line through M that is parallel to the side AB

Solution
For triangles ABC and NMC, $\angle ACB$ is the common angle Since MN is parallel to AB, the size of $\angle CMN$ is equal to the size of $\angle CBA$ (and the size of $\angle CNM$ is equal to the size of $\angle CAB$) Therefore triangle ABC and triangle NMC are similar. (AA similarity) By similarity $\frac{AC}{NC} = \frac{BC}{MC} = 2$ and so $\mathrm{AC} = 2\mathrm{NC}$. Hence $AN = NC$. Therefore triangles ANM and CNM are congruent (SAS) (equal sides AN and NC, common side NM and included angles $\angle ANM$ and $\angle CNM$ are both right angles). So $MA = MC$ So $MA = MB = MC$
Specific behaviours
✓✓ proves that Δ ABC and Δ NMC are similar(AA). ✓✓ proves that Δ ANM and Δ CNM are congruent (SAS) Note: Proofs which are generally correct but that contain some inappropriate reasoning will be awarded one mark

19. [6 marks]

(a) Given 8 students in a class are good debaters,

(i) in how many ways can a team of three be chosen from the 8 students? [1]

$^8C_3 = 56$ ✓

(ii) in how many ways can a team of three be chosen if one particular student is chosen as captain and another student of the 8 cannot attend the debate? [2]

$\underset{\substack{\uparrow \\ \text{Captain}}}{^1C_1} \; ^6C_2 = 15$ ✓✓

(b) Annie has a street stall. She sells T shirts in ten different colours but can only display six T shirts at a time.

(i) In how many ways can she display six different colour T-shirts in a line at the back of her stall? [1]

$^{10}P_6 = \frac{10!}{4!} = 151200$ ✓

Annie's football team wear yellow and blue.

(ii) In how many ways can Annie display six different coloured T-shirts with a yellow T-shirt next to a blue T-shirt? [2]

$\underset{\substack{\uparrow \\ \text{Yellow \& Blue.}}}{^2C_2} \times {}^8C_4 \times 5! \times \underset{\substack{\uparrow \\ \text{Yellow} \times \text{Blue.}}}{2!} = 16800$

✓✓